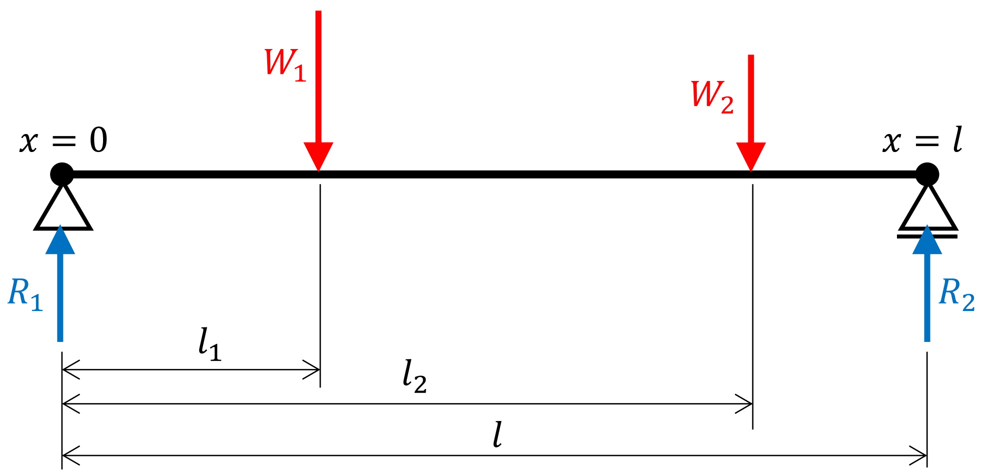
【両端支持】



力とモーメントの釣合い：

$$R_1 + R_2 = W_1 + W_2$$
$$R_2 l = W_1 l_1 + W_2 l_2$$

反力：

$$R_1 = W_1 \frac{l - l_1}{l} + W_2 \frac{l - l_2}{l}$$

$$R_2 = W_1 \frac{l_1}{l} + W_2 \frac{l_2}{l}$$

剪断力：

$$F = \begin{cases} R_1 = W_1 \dfrac{l - l_1}{l} + W_2 \dfrac{l - l_2}{l} & , \quad 0 \le x < l_1 \\ R_1 - W_1 = -W_1 \dfrac{l_1}{l} + W_2 \dfrac{l - l_2}{l} & , \quad l_1 < x < l_2 \\ R_1 - W_1 - W_2 = -W_1 \dfrac{l_1}{l} - W_2 \dfrac{l_2}{l} & , \quad l_2 < x \le l \end{cases}$$

モーメント：

$$M = \begin{cases} R_1 x & , \quad 0 \le x \le l_1 \\ R_1 x - W_1 (x - l_1) & , \quad l_1 \le x \le l_2 \\ R_1 x - W_1 (x - l_1) - W_2 (x - l_2) & , \quad l_2 \le x \le l \end{cases}$$

$$= \begin{cases} W_1 \dfrac{l - l_1}{l} x + W_2 \dfrac{l - l_2}{l} x & , \quad 0 \le x \le l_1 \\ W_1 \dfrac{l_1 (l - x)}{l} + W_2 \dfrac{l - l_2}{l} x & , \quad l_1 \le x \le l_2 \\ W_1 \dfrac{l_1 (l - x)}{l} + W_2 \dfrac{l_2 (l - x)}{l} & , \quad l_2 \le x \le l \end{cases}$$

撓み曲線：

$$\frac{d^2 v}{dx^2} = -\frac{M}{EI}$$

$0 \le x \le l_1$ のとき

$$\frac{dv}{dx} = -\frac{1}{EI}\left(\frac{R_1}{2} x^2 + C_1\right)$$

$$v = -\frac{1}{EI}\left(\frac{R_1}{6} x^3 + C_1 x\right)$$

$l_1 \le x \le l_2$ のとき

$$\frac{dv}{dx} = -\frac{1}{EI}\left(\frac{R_1 - W_1}{2} x^2 + W_1 l_1 x + C_2\right)$$

$$v = -\frac{1}{EI}\left(\frac{R_1 - W_1}{6} x^3 + \frac{W_1 l_1}{2} x^2 + C_2 x + C_3\right)$$

$l_2 \le x \le l$ のとき

$$\frac{dv}{dx} = -\frac{1}{EI}\left[\frac{R_1 - W_1 - W_2}{2} x^2 + (W_1 l_1 + W_2 l_2) x + C_4\right]$$

$$v = -\frac{1}{EI}\left(\frac{R_1 - W_1 - W_2}{6} x^3 + \frac{W_1 l_1 + W_2 l_2}{2} x^2 + C_4 x + C_5\right)$$

$x = l_1$ において，

$$\frac{R_1}{2} {l_1}^2 + C_1 = \frac{R_1 - W_1}{2} {l_1}^2 + W_1 {l_1}^2 + C_2$$

$$\therefore C_1 - C_2 = \frac{W_1}{2} {l_1}^2$$

$$\frac{R_1}{6}{l_1}^3 + C_1 l_1 = \frac{R_1 - W_1}{6}{l_1}^3 + \frac{W_1}{2}{l_1}^3 + C_2 l_1 + C_3$$

$$C_1 l_1 - C_2 l_1 - C_3 = \frac{W_1}{3}{l_1}^3$$

$$\therefore C_3 = (C_1 - C_2)l_1 - \frac{W_1}{3}{l_1}^3$$

$$\therefore C_3 = \frac{W_1}{2}{l_1}^3 - \frac{W_1}{3}{l_1}^3 = \frac{W_1}{6}{l_1}^3$$

$x = l_2$ において,

$$\frac{R_1 - W_1}{2}{l_2}^2 + W_1 l_1 l_2 + C_2 = \frac{R_1 - W_1 - W_2}{2}{l_2}^2 + (W_1 l_1 + W_2 l_2)l_2 + C_4$$

$$\therefore C_2 - C_4 = \frac{W_2}{2}{l_2}^2$$

$$\frac{R_1 - W_1}{6}{l_2}^3 + \frac{W_1}{2}l_1{l_2}^2 + C_2 l_2 + C_3 = \frac{R_1 - W_1 - W_2}{6}{l_2}^3 + \frac{W_1 l_1 + W_2 l_2}{2}{l_2}^2 + C_4 l_2 + C_5$$

$$\therefore C_2 l_2 + C_3 - C_4 l_2 - C_5 = \frac{W_2}{3}{l_2}^3$$

$$\therefore C_5 = (C_2 - C_4)l_2 + C_3 - \frac{W_2}{3}{l_2}^3 = \frac{W_1}{6}{l_1}^3 + \frac{W_2}{6}{l_2}^3 = \frac{W_1{l_1}^3 + W_2{l_2}^3}{6}$$

$x = l$ において,

$$\frac{R_1 - W_1 - W_2}{6}l^3 + \frac{W_1 l_1 + W_2 l_2}{2}l^2 + C_4 l + C_5 = 0$$

$$\therefore \frac{R_1 - W_1 - W_2}{6}l^3 + \frac{W_1 l_1 + W_2 l_2}{2}l^2 + C_4 l + \frac{W_1{l_1}^3 + W_2{l_2}^3}{6} = 0$$

$$\therefore -\frac{W_1 l_1 + W_2 l_2}{6}l^2 + \frac{W_1 l_1 + W_2 l_2}{2}l^2 + C_4 l + \frac{W_1{l_1}^3 + W_2{l_2}^3}{6} = 0$$

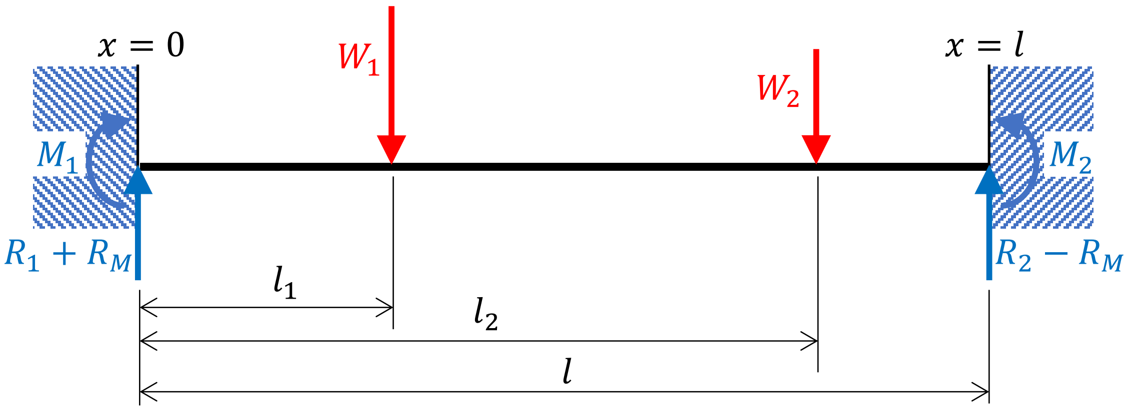
$$\therefore \frac{W_1 l_1 + W_2 l_2}{3}l^2 + C_4 l + \frac{W_1{l_1}^3 + W_2{l_2}^3}{6} = 0$$

$$\therefore C_4 = -\frac{W_1 l_1 l^2 + W_2 l_2 l^2}{3l} - \frac{W_1{l_1}^3 + W_2{l_2}^3}{6l} = -\frac{W_1 l_1(2l^2 + {l_1}^2) + W_2 l_2(2l^2 + {l_2}^2)}{6l}$$

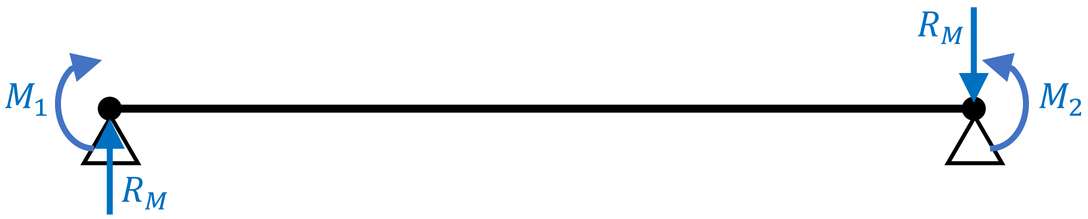
$$C_2 = \frac{W_2}{2}{l_2}^2 + C_4 = \frac{W_2}{2}{l_2}^2 - \frac{W_1 l_1(2l^2 + {l_1}^2) + W_2 l_2(2l^2 + {l_2}^2)}{6l}$$

$$C_1 = \frac{W_1}{2}{l_1}^2 + C_2 = \frac{W_1{l_1}^2 + W_2{l_2}^2}{2} - \frac{W_1 l_1(2l^2 + {l_1}^2) + W_2 l_2(2l^2 + {l_2}^2)}{6l}$$

【両端固定】



両端支持に以下を追加



追加モーメントの釣合い：

$$R_M l + M_1 = M_2$$

追加反力：

$$R_M = -\frac{M_1 - M_2}{l}$$

剪断力：

$$F = \begin{cases} R_1 + R_M = W_1\dfrac{l-l_1}{l} + W_2\dfrac{l-l_2}{l} - \dfrac{M_1-M_2}{l} , & 0 \le x < l_1 \\ R_1 - W_1 + R_M = -W_1\dfrac{l_1}{l} + W_2\dfrac{l-l_2}{l} - \dfrac{M_1-M_2}{l} , & l_1 < x < l_2 \\ R_1 - W_1 - W_2 + R_M = -W_1\dfrac{l_1}{l} - W_2\dfrac{l_2}{l} - \dfrac{M_1-M_2}{l} , & l_2 < x \le l \end{cases}$$

モーメント：

$$M = \begin{cases} (R_1+R_M)x + M_1 , & 0 \le x \le l_1 \\ (R_1+R_M)x - W_1(x-l_1) + M_1 , & l_1 \le x \le l_2 \\ (R_1+R_M)x - W_1(x-l_1) - W_2(x-l_2) + M_1 , & l_2 \le x \le l \end{cases}$$

$$
= \begin{cases} W_1 \dfrac{l-l_1}{l} x + W_2 \dfrac{l-l_2}{l} x - \dfrac{M_1 - M_2}{l} x + M_1 \quad , & 0 \le x \le l_1 \\ W_1 \dfrac{l_1(l-x)}{l} + W_2 \dfrac{l-l_2}{l} x - \dfrac{M_1 - M_2}{l} x + M_1 \ , & l_1 \le x \le l_2 \\ W_1 \dfrac{l_1(l-x)}{l} + W_2 \dfrac{l_2(l-x)}{l} - \dfrac{M_1 - M_2}{l} x + M_1 , & l_2 \le x \le l \end{cases}
$$

撓み曲線：

$$
\frac{d^2 v}{dx^2} = -\frac{M}{EI}
$$

$0 \le x \le l_1$ のとき

$$
\frac{dv}{dx} = -\frac{1}{EI}\left(\frac{R_1 + R_M}{2} x^2 + M_1 x + C_1\right)
$$

$$
v = -\frac{1}{EI}\left(\frac{R_1 + R_M}{6} x^3 + \frac{M_1}{2} x^2 + C_1 x\right)
$$

$l_1 \le x \le l_2$ のとき

$$
\frac{dv}{dx} = -\frac{1}{EI}\left[\frac{R_1 + R_M - W_1}{2} x^2 + (W_1 l_1 + M_1) x + C_2\right]
$$

$$
v = -\frac{1}{EI}\left(\frac{R_1 + R_M - W_1}{6} x^3 + \frac{W_1 l_1 + M_1}{2} x^2 + C_2 x + C_3\right)
$$

$l_2 \le x \le l$ のとき

$$
\frac{dv}{dx} = -\frac{1}{EI}\left[\frac{R_1 + R_M - W_1 - W_2}{2} x^2 + (W_1 l_1 + W_2 l_2 + M_1) x + C_4\right]
$$

$$
v = -\frac{1}{EI}\left(\frac{R_1 + R_M - W_1 - W_2}{6} x^3 + \frac{W_1 l_1 + W_2 l_2 + M_1}{2} x^2 + C_4 x + C_5\right)
$$

$x = 0$ において,

$$
\frac{dv}{dx} = -\frac{1}{EI} C_1 = 0
$$

$$
\therefore C_1 = 0
$$

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$x = l_1$ において，

$$\frac{R_1 + R_M}{2} {l_1}^2 + M_1 l_1 = \frac{R_1 + R_M - W_1}{2} {l_1}^2 + (W_1 l_1 + M_1) l_1 + C_2$$

$$\therefore C_2 = -\frac{W_1}{2} {l_1}^2$$

$$\frac{R_1 + R_M}{6} {l_1}^3 + \frac{M_1}{2} {l_1}^2 = \frac{R_1 + R_M - W_1}{6} {l_1}^3 + \frac{W_1}{2} {l_1}^3 + \frac{M_1}{2} {l_1}^2 + C_2 l_1 + C_3$$

$$\therefore C_3 = -\frac{W_1}{3} {l_1}^3 - C_2 l_1 = -\frac{W_1}{3} {l_1}^3 + \frac{W_1}{2} {l_1}^3 = \frac{W_1}{6} {l_1}^3$$

$x = l_2$ において，

$$\frac{R_1 + R_M - W_1}{2} {l_2}^2 + (W_1 l_1 + M_1) l_2 + C_2$$

$$= \frac{R_1 + R_M - W_1 - W_2}{2} {l_2}^2 + (W_1 l_1 + W_2 l_2 + M_1) l_2 + C_4$$

$$\therefore C_4 = -\frac{W_2}{2} {l_2}^2 + C_2 = -\frac{W_1 {l_1}^2 + W_2 {l_2}^2}{2}$$

$$\frac{R_1 + R_M - W_1}{6} {l_2}^3 + \frac{W_1 l_1 + M_1}{2} {l_2}^2 + C_2 l_2 + C_3$$

$$= \frac{R_1 + R_M - W_1 - W_2}{6} {l_2}^3 + \frac{W_1 l_1 + W_2 l_2 + M_1}{2} {l_2}^2 + C_4 l_2 + C_5$$

$$\therefore C_5 = -\frac{W_2}{3} {l_2}^3 + C_2 l_2 + C_3 - C_4 l_2 = -\frac{W_2}{3} {l_2}^3 - \frac{W_1}{2} {l_1}^2 l_2 + \frac{W_1}{6} {l_1}^3 + \frac{W_1 {l_1}^2 + W_2 {l_2}^2}{2} l_2$$

$$= \frac{W_1 {l_1}^3 + W_2 {l_2}^3}{6}$$

$x = l$ において，

$$\frac{R_1 + R_M - W_1 - W_2}{2} l^2 + (W_1 l_1 + W_2 l_2 + M_1) l + C_4 = 0$$

$$\therefore \frac{R_M}{2} l^2 + M_1 l = -\frac{R_1 - W_1 - W_2}{2} l^2 - (W_1 l_1 + W_2 l_2) l + \frac{W_1 {l_1}^2 + W_2 {l_2}^2}{2}$$

$$= \frac{W_1 l_1 + W_2 l_2}{2} l - (W_1 l_1 + W_2 l_2) l + \frac{W_1 {l_1}^2 + W_2 {l_2}^2}{2}$$

$$= -\frac{W_1 l_1 (l - l_1) + W_2 l_2 (l - l_2)}{2}$$

$$\frac{R_1 + R_M - W_1 - W_2}{6} l^3 + \frac{W_1 l_1 + W_2 l_2 + M_1}{2} l^2 + C_4 l + C_5 = 0$$

$$\therefore \frac{R_M}{6} l^3 + \frac{M_1}{2} l^2 = -\frac{R_1 - W_1 - W_2}{6} l^3 - \frac{W_1 l_1 + W_2 l_2}{2} l^2 - C_4 l - C_5$$

$$= \frac{W_1 l_1 + W_2 l_2}{6} l^2 - \frac{W_1 l_1 + W_2 l_2}{2} l^2 + \frac{W_1 {l_1}^2 + W_2 {l_2}^2}{2} l - \frac{W_1 {l_1}^3 + W_2 {l_2}^3}{6}$$

$$= -\frac{W_1 l_1 + W_2 l_2}{3} l^2 + \frac{W_1 {l_1}^2 + W_2 {l_2}^2}{2} l - \frac{W_1 {l_1}^3 + W_2 {l_2}^3}{6}$$

$$= -\frac{W_1 l_1 \left(2l^2 - 3l_1 l + {l_1}^2\right) + W_2 l_2 \left(2l^2 - 3l_2 l + {l_2}^2\right)}{6}$$

$$\frac{R_M}{6} l^3 = -\frac{W_1 l_1 (l - l_1) + W_2 l_2 (l - l_2)}{2} l + \frac{W_1 l_1 \left(2l^2 - 3l_1 l + {l_1}^2\right) + W_2 l_2 \left(2l^2 - 3l_2 l + {l_2}^2\right)}{3}$$

$$= \frac{W_1 l_1 \left(l^2 - 3l_1 l + 2{l_1}^2\right) + W_2 l_2 \left(l^2 - 3l_2 l + 2{l_2}^2\right)}{6}$$

$$\therefore R_M = \frac{W_1 l_1 \left(l^2 - 3l_1 l + 2{l_1}^2\right) + W_2 l_2 \left(l^2 - 3l_2 l + 2{l_2}^2\right)}{l^3}$$

$$-\frac{M_1}{2} l^2 = -\frac{W_1 l_1 (l - l_1) + W_2 l_2 (l - l_2)}{2} l + \frac{W_1 l_1 \left(2l^2 - 3l_1 l + {l_1}^2\right) + W_2 l_2 \left(2l^2 - 3l_2 l + {l_2}^2\right)}{2}$$

$$= \frac{W_1 l_1 (l - l_1)^2 + W_2 l_2 (l - l_2)^2}{2}$$

$$\therefore M_1 = -\frac{W_1 l_1 (l - l_1)^2 + W_2 l_2 (l - l_2)^2}{l^2}$$

$$M_2 = R_M l + M_1$$

$$= \frac{W_1 l_1 \left(l^2 - 3l_1 l + 2{l_1}^2\right) + W_2 l_2 \left(l^2 - 3l_2 l + 2{l_2}^2\right)}{l^2} - \frac{W_1 l_1 (l - l_1)^2 + W_2 l_2 (l - l_2)^2}{l^2} = -\frac{W_1 l_1 \left(l_1 l - {l_1}^2\right) + W_2 l_2 \left(l_2 l - {l_2}^2\right)}{l^2}$$

$$= -\frac{W_1 {l_1}^2 (l - l_1) + W_2 {l_2}^2 (l - l_2)}{l^2}$$